

DYNAMICS OF A MAGNETIC MONOPOLE IN MATTER, MAXWELL EQUATIONS IN DYONIC MATTER AND DETECTION OF ELECTRIC DIPOLE MOMENTS

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For a monopole, the analogue of the Lorentz equation in matter is shown to be $\mathbf{f} = g (\mathbf{H} - \mathbf{v} \times \mathbf{D})$. Dual-symmetric Maxwell equations, for matter containing hidden magnetic charges in addition to electric ones, are given. They apply as well to ordinary matter if the particles possess T-violating electric dipole moments. Two schemes of experiments for the detection of such moments in macroscopic pieces of matter are proposed.

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1. INTRODUCTION

The question of which classical macroscopic fields exert a force on a magnetic monopole of charge g in matter is still controversial [1]. For the static force, the formula

$$\mathbf{f} = g \mathbf{H}, \quad (1)$$

instead of $\mathbf{f} = g \mathbf{B}$, is generally accepted. However, for the velocity dependent force, there is no consensus between $\mathbf{f} = -g \mathbf{v} \times \mathbf{E}$ and $\mathbf{f} = -g \mathbf{v} \times \mathbf{D}$ (we use rationalized equations with $c = \varepsilon_0 = \mu_0 = 1$). A more general problem is to generalize the macroscopic Maxwell equations to the dual-symmetric matter. The atoms or molecules of such a matter would be made not only of electrically, but also of magnetically charged particles. Thus they can possess

- electric dipole moments coming from decentered electric charges as well as spinning magnetic charges
- magnetic moments coming from spinning electric charges as well as decentered magnetic charges.

After a rederivation and a discussion of Eq.(1), we will present below a consistent solution for the

velocity-dependent force and the dual-symmetric Maxwell equations in matter, using simple physical arguments. We will consider only isotropic matter and assume that its electric and magnetic polarizations \mathbf{P} and \mathbf{M} are linear in \mathbf{D} and \mathbf{B} (or \mathbf{E} and \mathbf{H}). It will appear that our equations can also take into account the electric dipole moments (e.d.m.) of the ordinary fermions generated by T-violating interactions, and we will propose two kinds of possible measurements of the e.d.m. in macroscopic matter.

2. STATIC FORCE ON A MONOPOLE IN MATTER.

If the force acting on a monopole in matter were $\mathbf{f} = g \mathbf{B}$, a monopole following a closed magnetic line of a permanent magnet could gain energy at each turn, providing a perpetual motion of the first species. This is an argument for choosing $\mathbf{f} = g \mathbf{H}$, the curl of which is zero for a static system.

One might object that the monopole can gain energy at each turn at the expense of the magnetic energy stored by the magnet and will eventually erase the magnetization of the metal. This is indeed what happens when a monopole is circulating through a superconducting loop : the varying flux

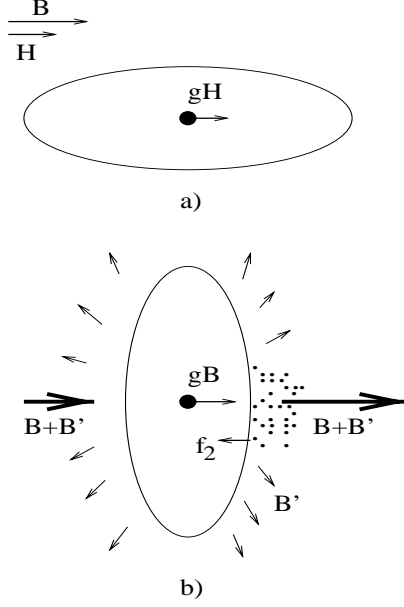


Figure 1: Force acting on a monopole in a ferrofluid. (a) elongated box parallel to the field (b) flattened box perpendicular to the field

of the monopole field through the loop produces a counter-electromotive force which damps the supercurrent. However, in the case of a ferroelectric annulus, the magnetized state has the lowest energy and the annulus cannot yield any energy to the monopole.

Another argument for (1) comes from the (gedanken) following experiment : Let us measure the force on a magnetic charge immersed in a ferrofluid. The latter is a practical realization of a liquid magnetizable matter. No static frictional force can perturb the measurement. We protect the monopole from the fluid by a waterproof box. This should not change the result ; anyway the physical monopole is probably dressed by a swarm of ordinary particles. In the absence of the monopole, we denote by $\mathbf{B} \equiv \mu\mathbf{H}$ the field outside the box and by $\mathbf{B}_{box} = \mathbf{H}_{box}$ the field inside the box. The fields coming from the monopole will be denoted by a prime. Let us consider two shapes of box (Fig.1) :

a) the box is elongated parallel to \mathbf{B} . Then $\mathbf{B}_{box} = \mathbf{H}$ and the measured force is $\mathbf{f} = g\mathbf{H}$.

b) the box is flattened perpendicular to \mathbf{B} . Then $\mathbf{B}_{box} = \mathbf{B}$. The force acting on the pole is $\mathbf{f}_1 = g\mathbf{B}$, which is different from case a). On the other hand, in front of the box the total field $\mathbf{B}^{tot} = \mathbf{B} + \mathbf{B}'$ is larger than behind. The magnetic grains of the ferrofluid are therefore attracted toward this region and build a hydrostatic pressure which pushes the box backwards. Quantitatively, the force acting on one grain of magnetic moment \vec{m} in the nonuniform field \mathbf{B}^{tot} is

$$f_i = m_j \partial_j B_i^{tot} = m_j \partial_i B_j^{tot}, \quad (2)$$

since $\nabla \times \mathbf{B}^{tot} = 0$ for a static system. Repeated indices are summed over. The resulting macroscopic force by unit volume is

$$\frac{dF_i}{d^3\mathbf{r}} = M_j^{tot} \partial_i B_j^{tot} = \frac{\chi}{2} \partial_i (\mathbf{B}^{tot} \cdot \mathbf{B}^{tot})$$

where $\mathbf{M}^{tot} = \mathbf{M} + \mathbf{M}' = \chi(\mathbf{B} + \mathbf{B}')$ is the magnetization density and $\chi = \frac{\mu-1}{\mu}$. This field of force builds the pressure

$$p = \frac{\chi}{2} \mathbf{B}^{tot} \cdot \mathbf{B}^{tot} = \frac{\chi}{2} (\mathbf{B}^2 + \mathbf{B}'^2) + \mathbf{M} \cdot \mathbf{B}'$$

The first two terms are symmetrical about the box and exert no net force on it. The last term gives

$$\mathbf{f}_2 = - \int (\mathbf{M} \cdot \mathbf{B}') d\mathbf{S} = - \int (\mathbf{B}' \cdot d\mathbf{S}) \mathbf{M} = -g\mathbf{M}.$$

Here $d\mathbf{S}$ is the vector representation of a surface element of the box and is directed outward. Permuting $d\mathbf{S}$ and \mathbf{M} was allowed because they are parallel in the region where \mathbf{B}' is important. The last equality comes from Gauss theorem for magnetic charges. Adding \mathbf{f}_1 and \mathbf{f}_2 one recovers the result (1) :

$$\mathbf{f} = \mathbf{f}_1 + \mathbf{f}_2 = g(\mathbf{B} - \mathbf{M}) = g\mathbf{H}. \quad (3)$$

Most probably, (1) can be generalized to any shape of box. Thus, the relevant field which drives a monopole in matter is \mathbf{H} . It is the field found in a parallel elongated cavity, as for the force $\mathbf{f} = e\mathbf{E}$ driving an electric charge inside a dielectric. *Mnemonic* : this kind of cavity allows the test charge to follow the force without touching the matter.

Eq.(1) allows trapping a (not too heavy) monopole in the pole of a permanent magnet, where the lines of $g\mathbf{H}$ converge from all directions. This would not be true for $g\mathbf{B}$.

Microscopic interpretation

Atomic magnetic dipoles are often pictured as microscopic loops of electrical current. Then \mathbf{B} appears as the average of the microscopic field \mathbf{b} over semi-macroscopic volumes sufficiently large compared to the atomic scale. The work of \mathbf{b} along a straight line L is therefore

$$g \int_L \mathbf{b} \cdot d\mathbf{l} = g \int_L \mathbf{B} \cdot d\mathbf{l}.$$

In contrast, the work along a line L' which avoids passing through the loops is

$$g \int_{L'} \mathbf{b} \cdot d\mathbf{l} = g \int_{L'} \mathbf{H} \cdot d\mathbf{l}.$$

Eq.(1) implies that the monopole avoids passing through the microscopic current loops or more likely that the loops move to "dodge" the monopole. This was of course the case with the ferrofluid, but in solid matter the atoms cannot escape from the monopole trajectory. Does it mean that (1) is false if the monopole goes through an atom? Not necessarily. At the approach of the pole, the electron wave functions are deformed and, if the monopole is sufficiently slow, they return adiabatically to the ground states. Thus no energy is exchanged between the monopole and the atom, as if the loop "dodges" the monopole.

3. VELOCITY-DEPENDENT FORCE

In vacuum, the analog of the Lorentz force for a moving monopole is $\mathbf{f} = -g\mathbf{v} \times \mathbf{E}$. Accordingly, a piece $d\mathbf{l}$ of wire carrying a current I^* of magnetic charges is subject to the dual Laplace force $d\mathbf{f} = -I^* d\mathbf{l} \times \mathbf{E}$. Following the ferrofluid example, we consider a wire protected by a waterproof tube in a liquid dielectric (Fig.2):

a) the tube is flattened perpendicular to $\mathbf{D} = \varepsilon\mathbf{E}$. Then $\mathbf{E}_{tube} = \mathbf{D}$ and the measured force is

$$d\mathbf{f} = -I^* d\mathbf{l} \times \mathbf{D}. \quad (4)$$

b) the tube is flattened parallel to $\mathbf{D} = \varepsilon\mathbf{E}$. Then $\mathbf{E}_{tube} = \mathbf{E}$. The force acting on the wire is $d\mathbf{f}_1 =$

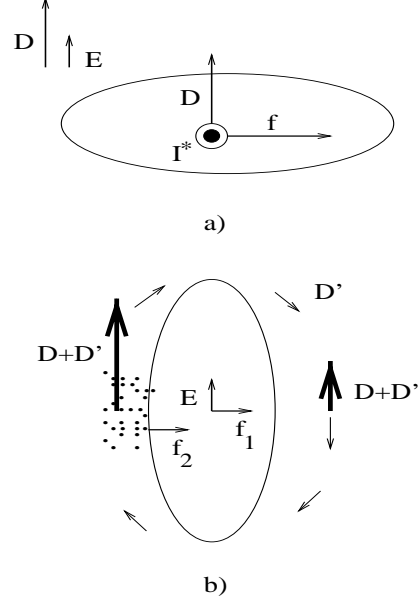


Figure 2: Force acting on a wire carrying the dual current I^* , in a dielectric liquid. The tube is flattened (a) perpendicular (b) parallel to the field.

$-I^* d\mathbf{l} \times \mathbf{E}$. On the other hand, on the right of the tube, the total field $\mathbf{D} + \mathbf{D}'$ is larger than on the left. The polar molecules are therefore attracted toward this region, building an excess of pressure which pushes the tube toward the left. Calculations like those between Eqs. (2) and (3) give the thrust $d\mathbf{f}_2 = -I^* d\mathbf{l} \times \mathbf{P}$, where \mathbf{P} is the macroscopic electric polarization. In total,

$$d\mathbf{f} = d\mathbf{f}_1 + d\mathbf{f}_2 = -I^* d\mathbf{l} \times (\mathbf{E} + \mathbf{P})$$

is equivalent to (4). Thus the field acting on a wire of magnetic current is \mathbf{D} . It is the field found in a perpendicular flattened cavity, as for the Laplace force $d\mathbf{f} = I d\mathbf{l} \times \mathbf{B}$ on an ordinary current in a magnetized matter. *Mnemonic*: this cavity allows the wire to follow the force without touching the matter.

For a moving monopole, (4) becomes

$$\mathbf{f} = -g\mathbf{v} \times \mathbf{D}. \quad (4')$$

Microscopic interpretation

In a dielectric, \mathbf{E} is the average of the microscopic field \mathbf{e} over volumes sufficiently large compared to the molecular scale. The work of \mathbf{e} when the wire sweeps a flat surface S is

$$I^* \iint_S \mathbf{e} \cdot d\mathbf{S} = I^* \iint_S \mathbf{E} \cdot d\mathbf{S}.$$

In contrast, the work of \mathbf{e} along a surface S' which avoids cutting the dipole molecules is

$$I^* \iint_{S'} \mathbf{e} \cdot d\mathbf{S} = I^* \iint_{S'} \mathbf{D} \cdot d\mathbf{S}.$$

Eq.(4) implies that a moving dual wire avoids cutting the dipole molecules, or that the molecules "dodge" the wire. This has a well-defined topological meaning. Let us recall however that this wire was introduced to make the problem time-independent. For a moving monopole, there is no swept surface and the topological interpretation is lost.

Gathering (1) and (4'), the total force on a magnetic charge is

$$\mathbf{f} = g (\mathbf{H} - \mathbf{v} \times \mathbf{D}). \quad (5)$$

This result does not take into account dissipation and holds only for sufficiently slow monopoles, such that atoms and molecules evolve adiabatically under the influence of the monopole field.

4. MAXWELL EQUATIONS IN DYONIC MATTER

We consider matter containing magnetic charges $\pm g$ bound in magnetically neutral molecules, in addition to ordinary particles. These molecules possess magnetic dipoles of the form $g\mathbf{r}$, building a macroscopic magnetic polarization \mathbf{p}^* (\mathbf{r} is the north-south charge separation). If they have spin, they also possess electric dipoles of the form $\gamma^*\mathbf{S}$ building a macroscopic electric polarization \mathbf{m}^* (γ^* is the "giroelectric" ratio). \mathbf{p}^* and \mathbf{m}^* are dual respectively to the polarization \mathbf{p} and the magnetization \mathbf{m} built by the ordinary particles. The dual-symmetric Maxwell equations for the space average of the microscopic fields are

$$\nabla \times \bar{\mathbf{b}} - \partial_t \bar{\mathbf{e}} = \mathbf{j} + \delta\mathbf{j}$$

$$\nabla \cdot \bar{\mathbf{e}} = \rho + \delta\rho$$

$$-\nabla \times \bar{\mathbf{e}} - \partial_t \bar{\mathbf{b}} = \mathbf{j}^* + \delta\mathbf{j}^*$$

$$\nabla \cdot \bar{\mathbf{e}} = \rho^* + \delta\rho^*.$$

Here (ρ, \mathbf{j}) is the external ordinary charge-current density and $(\delta\rho, \delta\mathbf{j})$ the induced one, given by

$$\delta\rho = -\nabla \cdot \mathbf{p}$$

$$\delta\mathbf{j} = \nabla \times \mathbf{m} + \partial_t \mathbf{p}$$

Similarly, for the magnetic charge analogues,

$$\delta\rho^* = -\nabla \cdot \mathbf{p}^*$$

$$\delta\mathbf{j}^* = -\nabla \times \mathbf{m}^* + \partial_t \mathbf{p}^*$$

From these equations, we can write the dual-symmetric Maxwell equations in matter :

$$\nabla \times \mathbf{H} - \partial_t \mathbf{D} = \mathbf{j}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$-\nabla \times \mathbf{E} - \partial_t \mathbf{B} = \mathbf{j}^*$$

$$\nabla \cdot \mathbf{B} = \rho^* \quad (6)$$

where $\mathbf{H} = \bar{\mathbf{b}} - \mathbf{m}$, $\mathbf{D} = \bar{\mathbf{e}} + \mathbf{p}$ as usual, but $\mathbf{E} = \bar{\mathbf{e}} - \mathbf{m}^*$ and $\mathbf{B} = \bar{\mathbf{b}} + \mathbf{p}^*$. We see that \mathbf{E} and \mathbf{B} can no more be interpreted as the spatial averages of the microscopic fields. In that sense they are no more "fundamental" than \mathbf{D} and \mathbf{H} . In fact the dual of \mathbf{E} is not \mathbf{B} but \mathbf{H} , whereas the dual of \mathbf{B} is $-\mathbf{D}$. The usual relations

$$\mathbf{D} = \mathbf{E} + \mathbf{P}, \quad \mathbf{B} = \mathbf{H} + \mathbf{M} \quad (7)$$

are recovered, defining

$$\mathbf{P} \equiv \mathbf{p} + \mathbf{m}^*, \quad \mathbf{M} \equiv \mathbf{m} + \mathbf{p}^*. \quad (8)$$

It means that the microscopic nature of the dipole is forgotten at the level of the macroscopic Maxwell equations. Only their long range fields in $1/r^3$ are relevant. As in ordinary matter, \mathbf{E} and \mathbf{H} are the fields found in elongated cavities parallel to the respective fields, whereas \mathbf{D} and \mathbf{B} are found in perpendicular flat cavities.

5. THE DYONIC PERMITTIVITY-PERMEABILITY MATRIX

We assume that the polarizations \mathbf{P} and \mathbf{M} respond linearly to the macroscopic fields \mathbf{D} and \mathbf{B} .

$$\begin{pmatrix} \mathbf{P} \\ \mathbf{M} \end{pmatrix} = [\chi] \begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = [\chi'] \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}, \quad (9)$$

with $[1 + \chi'] \equiv [1 - \chi]^{-1}$. In ordinary matter $\chi_{11} = \chi_e = \chi'_e/\varepsilon$, $\chi'_e \equiv \varepsilon - 1$; $\chi_{22} = \chi_m = \chi'_m/\mu$, $\chi'_m \equiv \mu - 1$, and $\chi_{12} = \chi_{21} = 0$. In a matter containing only one species of dyon (e, g) and antidyon ($-e, -g$) bound in polar molecules, $\mathbf{P} = \mathbf{p}$ and $\mathbf{M} = \mathbf{p}^*$ are linked by

$$\frac{\mathbf{p}}{e} = \frac{\mathbf{p}^*}{g}, \quad \frac{1}{e} \frac{\partial p_i}{\partial D_j} = \frac{1}{g} \frac{\partial p_i}{\partial B_j} \quad (10)$$

wherefrom

$$[\chi] = C^{te} \begin{pmatrix} e^2 & eg \\ eg & g^2 \end{pmatrix}. \quad (11)$$

An analogous matrix, with $e \leftrightarrow g$, is obtained with dipoles coming from spinning dyons ($\mathbf{m}/e = \mathbf{m}^*/g$). We note that $[\chi]$ and $[\chi']$ are symmetrical matrices. This remains true for a mixture of different species of molecules.

Thus, the usual relations $\mathbf{D} = \varepsilon \mathbf{E}$, $\mathbf{B} = \mu \mathbf{H}$ are replaced by

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = [1 - \chi] \begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix}, \quad (12)$$

The speed of light is

$$c = (\det[1 - \chi])^{\frac{1}{2}} \quad (c_{vac.} \equiv 1). \quad (13)$$

Whatever they come from, the nondiagonal elements of $[\chi]$ violate P- and T- symmetries, since they connect vector to pseudovectors. However PT is conserved.

6. ENERGY-MOMENTUM TENSOR

The various components of the energy-momentum tensor $\Theta^{\mu\nu}$ can be derived from energy and momentum conservation in simple physical systems. Let us suppose that the whole space is filled with dual-symmetric matter. To get Θ^{i0} (energy flow) and Θ^{ij}

(momentum flow) one considers a sandwich made of three slab-like regions of the z coordinate, $R_1 = [-a, 0]$, $R_2 = [0, b]$ and $R_3 = [b, b + a]$. R_1 carries uniform electric and magnetic charge-current densities, $\{\rho, \mathbf{j}; \rho^*, \mathbf{j}^*\}$ and R_3 carries the opposite densities, such that the fields vanish outside the sandwich. Solving (6) and (12) with appropriate $\rho, \mathbf{j}, \rho^*, \mathbf{j}^*$, any kind of uniform field configuration $\{\mathbf{E}, \mathbf{D}; \mathbf{H}, \mathbf{B}\}$ can be obtained in R_2 . These fields are linearly attenuated in R_1 and R_3 . In R_3 a power

$$\frac{dW}{dt d^3\mathbf{r}} = \mathbf{E} \cdot \mathbf{j} + \mathbf{H} \cdot \mathbf{j}^* \quad (14)$$

is dissipated and a force

$$\frac{d\mathbf{f}}{d^3\mathbf{r}} = \rho \mathbf{E} + \mathbf{j} \times \mathbf{B} + \rho^* \mathbf{H} - \mathbf{j}^* \times \mathbf{D} \quad (15)$$

is exerted per unit of volume. The same quantities per unit of area (integrated over z in R_3) give Θ^{z0} and Θ^{zi} in R_2 .

To get Θ^{00} (energy density) and Θ^{0i} (momentum density) one has to "rotate" the sandwich in the 4-dimensional space-time, replacing z by t and slabs by time-slices or "epoch" T_1, T_2, T_3 . During T_1 the (3-dimensional) space is filled with uniform current densities \mathbf{j} and \mathbf{j}^* , which progressively build uniform fields according (6) and (12). The second epoch is current-free and the uniform fields remain constant. The last epoch destroys the fields with opposite currents. Integrating (14) and (15) over t in T_3 give Θ^{00} and Θ^{0i} in T_2 . This method is detailed in [2]. One obtains

$$\Theta^{\mu\nu} = \begin{pmatrix} \frac{\mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B}}{2} & \mathbf{D} \times \mathbf{B} \\ \mathbf{E} \times \mathbf{H} & \Theta^{00} \delta^{ij} - D^i E^j - B^i H^j \end{pmatrix} \quad (16)$$

as in ordinary matter.

The Dirac condition in matter. One way to derive the Dirac condition between an electron and a monopole is to quantize the joint angular momentum of their fields which are

$$\mathbf{D} = \frac{e \mathbf{r}}{4\pi r^3}, \quad \mathbf{B} = \frac{g \mathbf{r}'}{4\pi r'^3}$$

where \mathbf{r} (resp. \mathbf{r}') is the distance from the charge (resp. the pole) to the observation point. According

to (15) the momentum density is $\Theta^{0i} = (\mathbf{D} \times \mathbf{B})_i$, from which one gets the angular momentum

$$\mathbf{J} = \iiint d^3\mathbf{r} \, \mathbf{r} \times (\mathbf{D} \times \mathbf{B}) = \frac{eg}{4\pi} \hat{\mathbf{n}} \quad (17)$$

where $\hat{\mathbf{n}}$ is the unit vector from the charge toward the pole. The usual Dirac condition $eg = 2n\pi\hbar$ is obtained from the quantization rule $\mathbf{J} \cdot \hat{\mathbf{n}} = n\hbar/2$. Note that if the momentum density were $\mathbf{E} \times \mathbf{H}$, as sometimes advocated (see the discussion in [3]), the Dirac condition in medium would not be consistent with that in vacuum.

7. APPLICATION TO THE SEARCH FOR AN ELECTRIC DIPOLE MOMENT

The dual-symmetric formalism applies as well to the case where the electron (or the nucleus) possesses an *electric dipole moment* (e.d.m.) $\vec{d} = \gamma^* \mathbf{S}$ in addition to the usual magnetic moment $\vec{m} = \gamma \mathbf{S}$. Then we have $\mathbf{m}/\gamma = \mathbf{m}^*/\gamma^*$ and a nondiagonal $[\chi]$ matrix element is generated, like with dyonic molecules (Eqs.10-11) :

$$[\chi] = \begin{pmatrix} \chi_e + \chi_m r^2 & \chi_m r \\ \chi_m r & \chi_m \end{pmatrix}, \quad (18)$$

where $r \equiv \gamma^*/\gamma$. $\chi_m = \frac{\mu-1}{\mu}$ comes from the spinning electrons and $\chi_e = \frac{\varepsilon-1}{\varepsilon}$ from polar molecules.

A nonzero χ_{12} may be generated in another way : the e.d.m. tends to align the spin of an electron along the internal electric field of a polar molecule. It couples \mathbf{m} to \mathbf{p} . Here we consider only the first mechanism.

Eq.(18) suggests two possible measurements of r :

a) In Fig.3a, a cylinder of magnetizable, but insulating material is immersed in a large magnetic field \mathbf{B}_0 . The inside field \mathbf{B} induces a small electric polarization $\mathbf{P} = \chi_m r \mathbf{B}$ and an electric field \mathbf{E} . If the cylinder is much broader than high, we have $\mathbf{B} = \mathbf{B}_0$, $\mathbf{D} \simeq 0$ and $\mathbf{E} \simeq -\mathbf{P}$. More generally one has

$$\langle \mathbf{E} \rangle = -x \chi_m r \mathbf{B}_0 \quad (19)$$

where the coefficient $x < 1$ depends on the container geometry. Let us take a cubic container of size L . Between the top and the bottom, we can measure

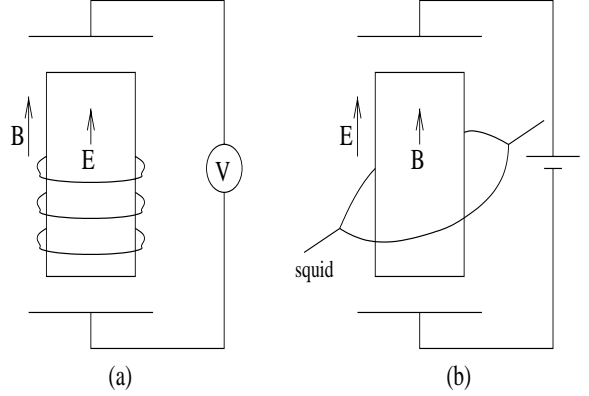


Figure 3: Scheme of e.d.m. search in macroscopic matter. (a) container in a magnetic field ; a small potential difference is measured with the voltmeter V. (b) container in an electric field ; a small magnetic flux is measured with a SQUID.

a potential difference $U = EL$. The ratio between the stored electrostatic energy $W = \frac{1}{2}\varepsilon \mathbf{E}^2 L^3$ and the magnetic one $W_0 = \frac{1}{2\mu} \mathbf{B}_0^2 L^3$ is

$$\frac{W}{W_0} = \varepsilon \mu (x \chi_m r)^2. \quad (20)$$

in terms of common units, we have

$$\frac{U}{\text{volt}} = 3 \cdot 10^8 x \chi_m r \frac{L}{\text{metre}} \cdot \frac{B_0}{\text{tesla}}$$

$$\frac{W}{\text{eV}} = 2.5 \cdot 10^{24} x^2 \varepsilon \chi_m^2 r^2 \left(\frac{L}{\text{metre}} \right)^3 \left(\frac{B_0}{\text{tesla}} \right)^2.$$

[useful relations are : 1 tesla = $3 \cdot 10^8$ volt/metre, $1 (\text{tesla})^2 (\text{metre})^3 = 0.8 \cdot 10^6$ joule, $1 \text{ eV} = 1.6 \cdot 10^{-19}$ joule $\simeq 10^4$ kelvin = $5 \cdot 10^6 \hbar/\text{metre}$ and $\gamma_e = e\hbar/m_e = 4 \cdot 10^{-11} \text{ e}\times\text{cm}$].

Let us assume $r = 10^{-16}$, which corresponds to an electron e.d.m. of $2 \cdot 10^{-27} \text{ e}\times\text{cm}$, $\varepsilon \sim 1$, $\chi_m \sim 0.5$ and $x \sim 0.5$. For a field of 1 tesla, and a cube of 1 meter, a potential difference of about $0.5 \cdot 10^{-8}$ volt is obtained. $W \sim 10^{-9} \text{ eV} \sim 10^{-5}$ kelvin. The voltmeter has to be cooled at least to this temperature to prevent thermal noise.

b) in Fig.3b the same container is put in a large electric field \mathbf{E}_0 . Using the second matrix of (9)

$$[\chi'] \simeq \begin{pmatrix} \varepsilon - 1 & \varepsilon \chi'_m r \\ \varepsilon \chi'_m r & \chi'_m \end{pmatrix}, \quad (18')$$

with $\chi'_m \equiv \mu - 1$, we predict a small magnetization $\mathbf{M} = \varepsilon \chi'_m r \mathbf{E}$. If the cylinder is much higher than broad, we have $\mathbf{E} \simeq \mathbf{E}_0$, $\mathbf{H} \simeq 0$ and $\mathbf{B} \simeq \mathbf{M}$. For a cubic container we assume

$$\mathbf{B} \simeq x \varepsilon \chi'_m r \mathbf{E}_0 \quad (21)$$

with $x \sim 0.5$. This field can be measured by a SQUID encircling the container. The phase shift of the wave function in one loop is

$$\begin{aligned} \varphi &= e L^2 B / \hbar \\ &= 0.5 \cdot 10^{12} \cdot x \varepsilon \chi'_m r \left(\frac{L}{\text{metre}} \right)^2 \frac{E_0}{10^5 \text{ volt/metre}}. \end{aligned}$$

This phase can be multiplied by a large number of turns around the cylinder. The ratio between the output (magnetostatic) energy W and the input (electric) one W_0 is still given by (20), but W and W_0 are typically 10^5 times smaller and the temperature must be much lower than in case a).

8. CONCLUSION

We have given arguments that the macroscopic fields acting on magnetic charges and currents are \mathbf{H} and \mathbf{D} . Comparing with electric charges and currents, one has a unified mnemonic principle : in each case, the acting field is the one found in a parallel-elongated (resp. flat-perpendicular) cavity in which a charge (resp. current wire) can follow the force without touching the medium. In a classical microscopic picture, a monopole avoids passing through the microscopic current loops and a dual current wire avoids cutting the dipole molecules. Quantum mechanically, it means that the perturbation of the atoms and molecules lying on the trajectory of the monopole is adiabatic. This should be the case at low enough velocity in a liquid. The monopole will be presumably accompanied by a swarm of atoms magnetically (or electrically, for a dyon) bound to it. In a solid, such a swarm could forbid the monopole to move without producing cracks.

The dual-symmetric Maxwell equations in matter are formally unchanged, but \mathbf{E} and \mathbf{B} can no more be interpreted as the spatial averages of the microscopic fields. The duality correspondance is $\mathbf{E} \rightarrow \mathbf{H}$ and $\mathbf{D} \rightarrow \mathbf{B}$. When dyons are present, or when ordinary particles possess electric dipole moments, ε and μ are replaced by a permittivity-permeability matrix $[1 - \chi]$ whose nondiagonal elements violate the P- and T- symmetries (but not PT). The energy momentum tensor is also unchanged. The usual Dirac condition $eg = 2n\pi \hbar$ is obtained provided the momentum density is $\mathbf{D} \times \mathbf{B}$. These results have been obtained under the hypothesis that \mathbf{P} and \mathbf{M} are linear in the fields.

As an application of the dual-symmetric formalism, two possible measurements of the electron e.d.m. have been suggested. They are at the limit of the present technological possibilities. However, mechanisms like the $\mathbf{m} - \mathbf{p}$ coupling in a polar molecule mentioned in Sect.7 might enhance the signal.

REFERENCES

1. Y. Hara, Electromagnetic force on a magnetic monopole, *Phys. Rev.*, 1985, v. A32, p.1002-1006
2. D. Fayolle, Dynamique d'un monopôle magnétique dans la matière, *LYCEN/9961*, july 1999, unpublished.
3. F.N.H. Robinson, Electromagnetic stress and momentum in matter, *Phys. Rep.*, 1975, v. C16, p. 313-354.